EJERCITACION

TRANSFORMADA Z

PARTE 1: TRANSFORMADA Z DIRECTA - PROPIEDADES

Ejercicio n° 1:

Halle, por definición, la transformada Z de las siguientes sucesiones definidas para $n \geq 0$ e indique en cada caso la región de convergencia correspondiente:

a) $x(n) = \left(\frac{1}{3}\right)^n$

b) $x(n) = \begin{cases} 1 & \text{si } n \text{ es par} \\ 0 & \text{si } n \text{ es impar} \end{cases}$

c) $x(n) = \begin{cases} 4^n & \text{si } n \text{ es par} \\ 3^{-n} & \text{si } n \text{ es impar} \end{cases}$

d) $x(n) = \begin{cases} 5 & \text{si } n \text{ es impar} \\ 3(2)^{-n} & \text{si } n \text{ es par} \end{cases}$

e) $x(n) = \begin{cases} 2 & \text{si } n = 1 \\ n & \text{si } 1 < n < 4 \\ 0 & \text{en otro caso} \end{cases}$

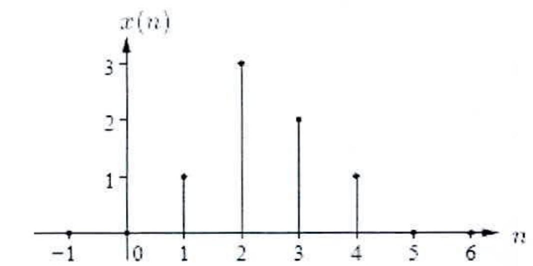
OPTATIVOS: f) $x(n) = \frac{1}{n+1}$ g) $x(n) = \frac{2^n}{n!}$

Recuerde: $\sum_{n=0}^{\infty} k\, r^n = \frac{K}{1-r}$ con $0 < r < 1$; $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$Ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n+1}\frac{x^n}{n} \quad \text{con } x \in (-1\,;\,1]$$

Ejercicio n° 2:

Dada la siguiente secuencia finita, halle la transformada Z de la misma.



Ejercicio n° 3:

Utilizando la tabla y las propiedades, halle la transformada Z de las siguientes sucesiones:

a) $x(n) = 2^n + 3\left(\frac{1}{2}\right)^n$; $n \geq 0$ b) $x(n) = u(n-2)$; $n \geq 2$ c) $x(n) = 3^{n+1}$; $n \geq 0$

Ejercicio n° 4:

Indique el valor de verdad de las siguientes proposiciones, justificando correctamente:

a) La región de convergencia de la transformada Z de $a_n = \left(\frac{1}{3}\right)^n \quad \forall\, n \in N$ es: $|z| > 3$

b) La transformada Z de $x(n) = \begin{cases} 4 & \text{si } n \text{ es par} \\ 5^{-n} & \text{si } n \text{ es impar} \end{cases}$ es: $X(z) = \dfrac{4z^2}{z^2-1} + \dfrac{5\,z}{25z^2-1}$

c) Dada la secuencia $x(n) = \begin{cases} 3 & \text{si } n \text{ es multiplo de } 5 \\ (-2)^n & \text{si } n \text{ no es multiplo de } 5 \end{cases}$, la región de convergencia de su transformada Z es: $|z|>2$

PARTE 2: ANTITRANSFORMADA Z

Ejercicio n° 5:

Halle las antitransformadas Z de las siguientes funciones:

a) $X(z) = \dfrac{3z(z-3)}{z^2-5z+4}$ b) $X(z) = \dfrac{2\,z^2}{z^2-9}$ c) $X(z) = \dfrac{z}{z^2+z-2}$

d) $X(z) = \operatorname{senh}(\frac{2}{z})$ e) $X(z) = z\,(e^{\frac{1}{z}}-1)$ f) $X(z) = \left(\dfrac{1}{z-1}\right)^2$

g) $X(z) = \dfrac{2z^2+z}{(z-2)^2(z-1)}$ h) $X(z) = \dfrac{-z^2+4z}{(z-2)(z-1)^2}$ i) $X(z) = \dfrac{4\,z^3-14\,z^2+10z}{(z-3)(z^2-4z+4)}$

Recuerde: $\cosh(x) = \dfrac{e^x+e^{-x}}{2}$; $\operatorname{senh}(x) = \dfrac{e^x-e^{-x}}{2}$

PARTE 3: ECUACIONES EN DIFERENCIAS

☑ Ejercicio n° 6:

Resuelva las siguientes ecuaciones en diferencias de orden 1 utilizando la transformada Z:

a) $a_{n+1} - 5\, a_n = 12$ con $a_0 = -1$

b) $a_{n+1} - 2\, a_n = 2$ con $a_0 = 3$

c) $a_{n+1} - 2\, a_n = 2 \bullet 3^n$ con $a_0 = 2$

d) $a_{n+1} = 2\, a_n + 3\, n - 1$ con $a_0 = 0$

e) $a_{n+1} - 4\, a_n = 4\, (1-n)\, 2^n$ con $a_0 = 3$

☑ Ejercicio n° 7:

Resuelva las siguientes ecuaciones en diferencias de orden 2 utilizando la transformada Z:

a) $a_{n+2} + 3\, a_{n+1} + 2\, a_n = 3^n$ con $a_0 = 0 \wedge a_1 = 1$

b) $a_{n+2} + 4\, a_{n+1} + 4\, a_n = 7$ con $a_0 = 1 \wedge a_1 = 2$

c) $x(n+2) - 4\, x(n+1) + 3\, x(n) = -2 \wedge x(0) = 7 \wedge x(1) = 12$

d) $x(n+2) - 4\, x(n+1) + 4\, x(n) = 4 \bullet 3^n \wedge x(0) = 4 \wedge x(1) = 14$

e) $x(n+2) - 6\, x(n+1) + 9\, x(n) = 5 \bullet 2^n \wedge x(0) = 5 \wedge x(1) = 13$

f) $a_{n+2} = a_{n+1} + a_n$ con $a_0 = 0 \wedge a_1 = 1$

g) $a_{n+2} - a_n = \text{sen}(\frac{n\pi}{2})$ con $a_0 = 1 \wedge a_1 = 1$

☑ Ejercicio n° 8:

Para las siguientes ecuaciones en diferencias de orden 2 escriba la X(z) y la x(n) correspondiente a cada una:

a) $x(n+2) - 6\, x(n+1) + 9\, x(n) = 100\, (-2)^n$ con $x(0) = 4 \wedge x(1) = -5$

b) $x(n+2) - 4\, x(n+1) + 4\, x(n) = 4^{n+1}$ con $x(0) = 1 \wedge x(1) = 10$

c) $a_{n+2} - 2\, a_{n+1} - 3\, a_n = 24 \bullet 3^n + 12 \bullet 5^n$ con $a_0 = 1 \wedge a_1 = 11$

EJERCICIOS PARA MARCAR LA RESPUESTA CORRECTA, tomados en Finales:

☑ Ejercicio n° 9:

Marcar la única respuesta correcta de cada ítem:

1.	La transformada Z de: $x(n) = \begin{cases} 0 & \text{si n es par} \\ 2\,(-3)^{-n} & \text{si n es impar} \end{cases}$ es: a) $X(z) = \dfrac{-6z}{9z^2-1}$ b) $X(z) = \dfrac{2z}{z+3}$ c) $X(z) = \dfrac{2z}{z^2+9}$ d) n.a.
2.	La transformada Z de $x(n) = \begin{cases} 0 & \text{si n es par} \\ 3 \bullet 2^n & \text{si n es impar} \end{cases}$ es: a) $X(z) = \dfrac{3z}{z-2}$ b) $X(z) = \dfrac{3z}{z^2-4}$ c) $X(z) = \dfrac{3z^2}{z^2-4}$ d) $X(z) = \dfrac{6z}{z^2-4}$ e) n.a.
3.	Sea $x(n) = n$ para $0 \leq n \leq 3$ $\wedge$ $x(n) = 0$ para $n \geq 4$, entonces su transformada Z es: a) $Z[x(n)] = \dfrac{z}{(z-1)^2}$ b) $Z[x(n)] = \dfrac{z+2z+3z}{(z-3)^2}$ c) $Z[x(n)] = \dfrac{z^2+2z+3}{z^3}$ d) n.a.
4.	Sean las sucesiones a_n y b_n entonces se cumple: a) $Z[a_n \bullet b_n] = Z[a_n] \bullet Z[b_n]$ b) $Z[a_n^2] = Z[a_n]^2$ c) $Z[a_n - k\, b_n] = Z[a_n] - k\, Z[b_n]$ d) $Z[a_{n+1}] = z\, Z[a_n] - a_0$ e) n.a.
5.	La antitransformada Z de $X(z) = \dfrac{z(2z+3)}{z^2-7z+6}$ es: a) $x(n) = 3\,(-6)^n + 1$ b) $x(n) = -3\,(-6)^n + (-1)^n$ c) $x(n) = 3 \bullet 6^n - 1$ d) n.a.
6.	La antitransformada Z de $X(z) = \dfrac{12z}{z^2-9}$ es: a) $x(n) = 12 \cdot 3^n$ b) $x(n) = 12\,[(3)^2]^n$ c) $x(n) = 12 \cdot (-3)^n + 12 \cdot 3^n$ d) $x(n) = 4 \cdot 3^n$ si n es impar e) n.a.

2018

TRANSFORMADA Z

Matemática Superior

PARTE 1: transformada Z directa - propiedades

① Halle, por definición, la transformada Z de las sig. sucesiones definidas para $n \geq 0$ e indique en cada caso la región de convergencia correspondiente:

a) $x(n) = \left(\frac{1}{3}\right)^n$

$$Z[x(n)] = \sum_{n=0}^{\infty} x(n)\, z^{-n} = X(z)$$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n = \frac{1}{1-\frac{1}{3z}} = \frac{1}{\frac{3z-1}{3z}} = \boxed{\frac{3z}{3z-1} = X(z)}, \quad |z| > 1/3$$

b) $x(n) = \begin{cases} 1 & \text{si } n \text{ es par} \\ 0 & \text{si } n \text{ es impar} \end{cases}$ $\quad n \text{ par} = 2k$

$$Z[x(n)] = \sum_{n=0}^{\infty} x(n)\, z^{-n} = \sum_{k=0}^{\infty} \left(\frac{1}{z}\right)^{2k} = \sum_{k=0}^{\infty} \left(\left(\frac{1}{z}\right)^2\right)^k = \sum_{k=0}^{\infty} \left(\frac{1}{z^2}\right)^n =$$

$$= \frac{1}{1-\frac{1}{z^2}} = \frac{1}{\frac{z^2-1}{z^2}} = \boxed{\frac{z^2}{z^2-1} = X(z)} \quad |z^2| > 1 \rightarrow \boxed{|z| > 1}$$

c) $x(n) = \begin{cases} 4^n & \text{si } n \text{ es par} \\ 3^{-n} & \text{si } n \text{ es impar} = \left(\frac{1}{3}\right)^n \end{cases}$

$$Z[x(n)] = X(z) = \sum_{k=0}^{\infty} \left[\left(\frac{4}{z}\right)^{2k} + \left(\frac{1}{3z}\right)^{2k+1}\right] = \sum_{k=0}^{\infty} \left[\overbrace{\left(\frac{16}{z^2}\right)^k}^{|z|>4} + \frac{1}{3z}\overbrace{\left(\frac{1}{9z^2}\right)^k}^{|z|>1/3}\right] =$$

$$= \frac{1}{1-\frac{16}{z^2}} + \frac{1}{3z}\cdot\frac{1}{1-\frac{1}{9z^2}} = \frac{1}{\frac{z^2-16}{z^2}} + \frac{1}{\cancel{3z}}\cdot\frac{1}{\frac{9z^2-1}{9z^2}} =$$

$$= \boxed{\frac{z^2}{z^2-16} + \frac{3z}{9z^2-1} = X(z)}$$

d) $x(n) = \begin{cases} 5 & \text{si } n \text{ es impar} \\ 3(2)^{-n} & \text{si es par} \end{cases}$

$$Z[x(n)] = X(z) = \sum_{k=0}^{\infty} \left[5\left(\frac{1}{z}\right)^{2k+1} + 3\left(\frac{1}{2z}\right)^{2k}\right] = \sum_{k=0}^{\infty} \frac{5}{z}\left(\frac{1}{z^2}\right)^k + 3\left(\frac{1}{4z^2}\right)^k =$$

mayor

$|z| > 1$ $\quad \left|\frac{1}{4z^2}\right| < 1 \rightarrow |4z^2| > 1$ $\quad |z| > 1/2$

$$= \frac{5}{\cancel{z}}\cdot\frac{z^{\cancel{2}}}{z^2-1} + 3\cdot\frac{4z^2}{4z^2-1} = \boxed{\frac{5z}{z^2-1} + \frac{12z^2}{4z^2-1} = X(z)} \quad |z| > 1$$

Syluma

e) $x(n) = \begin{cases} 2 & \text{si } n=1 \\ n & \text{si } 1<n<4 \\ 0 & \text{en otro caso} \end{cases}$ $\rightarrow n=2,\ n=3$

$$Z[x(n)] = \frac{2}{z} + \frac{2}{z^2} + \frac{3}{z^3} = \boxed{\frac{2z^3+2z+3}{z^3} = X(z)} \qquad |z| \neq 0$$ ✓

f) $x(n) = \frac{1}{n+1}$ (recordar que $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + \dots + (-1)^{n+1}\frac{x^n}{n}$; $-1<x<1$)

$$Z[x(n)] = \sum_{n=0}^{\infty} \frac{1}{n+1} \cdot \frac{1}{z^n} = z \ln\left(1 - \frac{1}{z}\right) \qquad |z|>1$$

g) $x(n) = \frac{2^n}{n!}$ (Recordar que $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$)

$$Z[x(n)] = X(z) = \sum_{n=0}^{\infty} \frac{2^n}{z^n} \cdot \frac{1}{n!} = \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n \frac{1}{n!} = \boxed{e^{\frac{2}{z}} = X(z)} \qquad |z|>2$$

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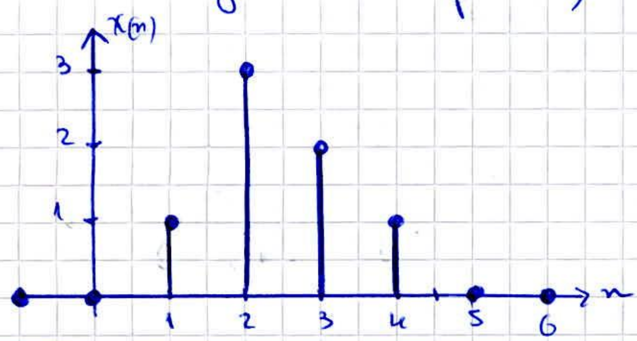
② Dado la sig. secuencia finita, halle la transformada Z de la misma:



$x(n) = \begin{cases} 1 & \text{si } n=1 \vee n=4 \\ 3 & \text{si } n=2 \\ 2 & \text{si } n=3 \\ 0 & \text{en otro caso} \end{cases}$

$$X(z) = \frac{1}{z^1} + \frac{1}{z^4} + \frac{3}{z^2} + \frac{2}{z^3} = \boxed{\frac{z^3+1+3z^2+2z}{z^4} = X(z)}$$ ✓

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③ Utilizando la tabla y las propiedades, halle la transformada Z de las sig. sucesiones:

a) $x(n) = 2^n + 3\left(\frac{1}{2}\right)^n$; $n \geq 0$ $\quad X(z) = \frac{z}{z-2} + 3 \cdot \frac{z}{z-1/2} = \frac{z}{z-2} + \frac{3z}{\frac{2z-1}{2}} =$

$$= \frac{z}{z-2} + \frac{6z}{2z-1} = \frac{2z^2 - z + 6z^2 - 12z}{(z-2)(2z-1)} = \boxed{\frac{8z^2-13z}{(z-2)(2z-1)} = X(z)} \qquad |z|>2$$ ✓

b) $x(n) = u(n-2)$; $n \geq 2$

$$X(z) = z^{-2}\, U(z) = \frac{1}{z^2} \cdot \frac{z}{z-1} = \boxed{\frac{1}{z(z-1)} = X(z)}$$ ✓

c) $x(n) = 3^{n+1} = 3 \cdot 3^n$

$$\boxed{X(z) = 3 \cdot \frac{z}{z-3}}$$ ✓

(4) Indique el valor de verdad de las sig. proposiciones, justificando correctamente:

a) La región de convergencia de la transformada Z de $a_n = \left(\frac{1}{3}\right)^n \ \forall n \in \mathbb{N}$ es $|z| > 3$

F . $|z| > 1/3$ ✓

b) La transformada Z de $x(n) = \begin{cases} 4 & \text{si } n \text{ es par} \\ 5^n & \text{si } n \text{ es impar} \end{cases}$ es $X(z) = \frac{4z^2}{z^2-1} + \frac{5z}{25z^2-1}$ V

• la profesora dijo que es 5^{-n}

$$X(z) = \sum_{k=0}^{\infty} \left[4\left(\frac{1}{z}\right)^{2k} + \left(\frac{1}{5z}\right)^{2k+1} \right] = 4\sum_{k=0}^{\infty} \left(\frac{1}{z^2}\right)^k + \frac{1}{5z}\sum_{k=0}^{\infty}\left(\frac{1}{25z^2}\right)^k = \frac{4z^2}{z^2-1} + \frac{1}{\cancel{5z}} \cdot \frac{\overset{5}{\cancel{25}}z^2}{25z^2-1}$$

c) Dada la secuencia $x(n) = \begin{cases} 3 & \text{si } n \text{ es múltiplo de } 5 \\ (-2)^n & \text{si } n \text{ NO es " de } 5 \end{cases}$, la región de convergencia de su transf. Z es $|z| > 2$ V ✓

$$X(z) = \sum_{k=1}^{\infty} 3\left(\frac{1}{z}\right)^{5k} + \left(\frac{-2}{z}\right)^{5k+1} + \left(\frac{-2}{z}\right)^{5k+2} + \left(\frac{-2}{z}\right)^{5k+3} + \left(\frac{-2}{z}\right)^{5k+4} \qquad |z| > 2$$

PARTE 2: Antitransformada Z

(5) Halle las antitransformadas Z de las sig. funciones:

a) $X(z) = \frac{3z(z-3)}{z^2-5z+4} = z\left[\frac{3(z-3)}{(z-4)(z-1)}\right] = z\left[\frac{A}{z-4} + \frac{B}{z-1}\right] = z\left[\frac{A(z-1)+B(z-4)}{(z-4)(z-1)}\right]$

$\rightarrow \begin{cases} A+B = 3 \\ -A-4B = -9 \end{cases} \rightarrow \begin{matrix} A = 1 \\ B = 2 \end{matrix} \rightarrow X(z) = \frac{z}{z-4} + \frac{2z}{z-1} \rightarrow \boxed{x(n) = 4^n + 2}$ ✓

b) $X(z) = \frac{2z^2}{z^2-9} = z\left[\frac{2z}{z^2-9}\right] = z\left[\frac{2z}{(z-3)(z+3)}\right] = z\left[\frac{A}{z-3} + \frac{B}{z+3}\right] = z\left[\frac{A(z+3)+B(z-3)}{z^2-9}\right]$

$\begin{cases} A+B = 2 \\ 3A-3B = 0 \end{cases} \rightarrow \begin{matrix} A = 1 \\ B = 1 \end{matrix} \rightarrow X(z) = \frac{z}{z-3} + \frac{z}{z+3} \rightarrow \boxed{x(n) = (-3)^n + 3^n}$

→ si n es impar → $-3+3=0$
→ si n es par → $3^n + 3^n = 2 \cdot 3^n$ → $\boxed{x(n) = \begin{cases} 0 & \text{si } n \text{ es impar} \\ 2 \times 3^n & \text{si } n \text{ es par} \end{cases}}$ ✓

c) $X(z) = \frac{z}{z^2+z-2} = \frac{z}{(z-1)(z+2)} = z\left[\frac{A}{z-1} + \frac{B}{z+2}\right] = z\left[\frac{A(z+2)+B(z-1)}{(z-1)(z+2)}\right]$

$\begin{cases} A+B = 0 \\ 2A-B = 1 \end{cases} \rightarrow \begin{matrix} A = 1/3 \\ B = -1/3 \end{matrix} \rightarrow X(z) = \frac{1/3\, z}{z-1} - \frac{1/3\, z}{z+2} \rightarrow \boxed{x(n) = \frac{1}{3} - \frac{1}{3}(-2)^n}$ ✓

d) $X(z) = \operatorname{senh}\left(\frac{2}{z}\right)$ $\qquad \operatorname{senh}(x) = \frac{e^{x} - e^{-x}}{2}$

$$X(z) = \frac{e^{\frac{2}{z}} - e^{-\frac{2}{z}}}{2}$$, ej 1.g: $x(n) = \frac{2^n}{n!} \rightarrow X(z) = e^{\frac{2}{z}}$

$$X(z) = \frac{e^{\frac{2}{z}}}{2} + \frac{e^{-\frac{2}{z}}}{-2} \rightarrow x(n) = \frac{2^{n-1}}{n!} + \frac{(-2)^{n-1}}{n!} \rightarrow \boxed{x(n) = \begin{cases} \frac{2^n}{n!} & \text{si } n \text{ impar} \\ 0 & \text{si } n \text{ par} \end{cases}}$$

e) $X(z) = z\left(e^{\frac{1}{z}} - 1\right) = z e^{\frac{1}{z}} - z$ $\qquad x(n) = \frac{1}{(n+1)!}$

f) $X(z) = \left(\frac{1}{z-1}\right)^2 = \frac{1}{(z-1)^2}$

$Z^{-1}\left[\frac{z}{(z-1)^2}\right] = n \rightarrow Z^{-1}\left[z^{-1}\frac{z}{(z-1)^2}\right] \rightarrow x(n) = n-1$

g) $$X(z) = \frac{2z^2 + z}{(z-2)^2(z-1)} = \frac{z(2z+1)}{(z-2)^2(z-1)} = z\left[\frac{A}{(z-2)} + \frac{B}{(z-2)^2} + \frac{C}{z-1}\right] =$$

$$= z\left[\frac{A\overbrace{(z-2)(z-1)}^{z^2-3z+2} + B(z-1) + C(z^2-4z+4)}{(z-2)^2(z-1)}\right] \rightarrow \begin{cases} A + C = 0 \\ -3A + B - 4C = 2 \\ 2A - B + 4C = 1 \end{cases} \rightarrow \begin{matrix} A = -3 \\ B = 5 \\ C = 3 \end{matrix}$$

$$X(z) = \frac{-3z}{z-2} + \frac{2.5z}{2(z-2)^2} + \frac{3}{z-1} \rightarrow \boxed{x(n) = -3 \cdot 2^n + \frac{5}{2} n \cdot 2^n + 3}$$

i) $$X(z) = \frac{4z^3 - 14z^2 + 10z}{(z-3)(z^2 - 4z + 4)} = z\left[\frac{4z^2 - 14z + 10}{(z-3)(z-2)^2}\right] = z\left[\frac{A}{(z-3)} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2}\right] =$$

$$= z\left[\frac{A(z^2 - 4z + 4) + B\overbrace{(z-3)(z-2)}^{z^2-5z+6} + C(z-3)}{(z-3)(z-2)^2}\right] \rightarrow \begin{cases} A + B = 4 \\ -4A - 5B + C = -14 \\ 4A + 6B - 3C = 10 \end{cases} \quad \begin{matrix} A = 4 \\ B = 0 \\ C = 2 \end{matrix}$$

$$X(z) = \frac{4z}{z-3} + \frac{2z}{(z-2)^2} \rightarrow \boxed{x(n) = 4 \times 3^n + n\, 2^n}$$

PARTE 3: Ecuaciones en diferencias

⑥ Resuelva las sig. ecuaciones en diferencias de orden 1 utilizando la transformada Z:

a) $a_{n+1} - 5a_n = 12$ con $a_0 = -1$

$x(n+1) - 5\,x(n) = 12 \qquad x(0) = -1$

$z\left[X(z) - \overset{-1}{x(0)}\right] - 5\,X(z) = \dfrac{12z}{z-1}$

$X(z)(z-5) = \dfrac{12z}{z-1} - z = \dfrac{12z - z^2 + z}{z-1} = \dfrac{-z^2+13z}{z-1}$

$X(z) = z\left[\dfrac{-z+13}{(z-1)(z-5)}\right] = z\left[\dfrac{A(z-5)+B(z-1)}{(z-1)(z-5)}\right] \rightarrow \begin{cases} A+B = -1 \\ -5A-B = 13 \end{cases} \rightarrow \begin{matrix} A=-3 \\ B=2 \end{matrix}$

$X(z) = \dfrac{-3z}{(z-1)} + \dfrac{2z}{z-5} \rightarrow \boxed{x(n) = -3 + 2 \times 5^n}$ ✓

b) $a_{n+1} - 2a_n = 2$ con $a_0 = 3$

$x(n+1) - 2\,x(n) = 2$ con $x(0) = 3$

$z\left[X(z) - \overset{3}{x(0)}\right] - 2\,X(z) = \dfrac{2z}{z-1}$

$X(z)(z-2) = \dfrac{2z}{z-1} + 3z = \dfrac{2z + 3z^2 - 3z}{z-1} = \dfrac{3z^2 - z}{z-1}$

$X(z) = z\left[\dfrac{3z-1}{(z-1)(z-2)}\right] = z\left[\dfrac{A(z-2)+B(z-1)}{(z-1)(z-2)}\right] \rightarrow \begin{cases} A+B = 3 \\ -2A-B = -1 \end{cases} \rightarrow \begin{matrix} A=-2 \\ B=5 \end{matrix}$

$X(z) = \dfrac{-2z}{z-1} + \dfrac{5z}{z-2} \rightarrow \boxed{x(n) = -2 + 5 \times 2^n}$ ✓

c) $a_{n+1} - 2a_n = 2 \cdot 3^n$ con $a_0 = 2$

$x(n+1) - 2\,x(n) = 2 \times 3^n$ con $x(0) = 2$

$z\left[X(z) - \overset{2}{x(0)}\right] - 2\,X(z) = 2 \cdot \dfrac{z}{z-3}$

$X(z)(z-2) = \dfrac{2z}{z-3} + 2z = \dfrac{2z + 2z^2 - 6z}{z-3} = \dfrac{2z^2 - 4z}{z-3} = z\left[\dfrac{2z-4}{z-3}\right]$

$X(z) = z\left[\dfrac{2z-4}{(z-2)(z-3)}\right] = z\left[\dfrac{A(z-3)+B(z-2)}{(z-2)(z-3)}\right] \rightarrow \begin{cases} A+B = 2 \\ -3A-2B = -4 \end{cases} \rightarrow \begin{matrix} A=0 \\ B=2 \end{matrix}$

$X(z) = \dfrac{2z}{z-3} \rightarrow \boxed{x(n) = 2 \times 3^n}$ ✓

d) $a_{n+1} = 2a_n + 3n - 1$ con $a_0 = 0$

$$x(n+1) - 2x(n) = 3n - 1 \qquad x(0) = 0$$

$$z\left[X(z) - \overset{0}{x(0)}\right] - 2X(z) = 3\frac{z}{(z-1)^2} - \frac{z}{z-1} = z\left[\frac{3}{(z-1)^2} - \frac{1}{z-1}\right]$$

$$X(z)(z-2) = z\left[\frac{3-(z-1)}{(z-1)^2}\right] = z\left[\frac{-z+4}{(z-1)^2}\right]$$

$$X(z) = z\left[\frac{-z+4}{(z-1)^2(z-2)}\right] = z\left[\frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z-2}\right] = z\left[\frac{A\overbrace{(z-1)(z-2)}^{z^2-3z+2} + B(z-2) + C\overbrace{(z-1)^2}^{z^2-2z+1}}{(z-1)^2(z-2)}\right]$$

$$\begin{cases} A + C = 0 \\ -3A + B - 2C = -1 \\ 2A - 2B + C = 4 \end{cases} \rightarrow \begin{matrix} A = -2 \\ B = -3 \\ C = 2 \end{matrix} \qquad X(z) = \frac{-2z}{z-1} - \frac{3z}{(z-1)^2} + \frac{2}{z-2}$$

$$\boxed{x(n) = -2 - 3n + 2^{n+1}}$$

e) $a_{n+1} - 4a_n = 4(1-n)2^n$ con $a_0 = 3$

$$x(n+1) - 4x(n) = 4 \times 2^n - 4n\,2^n \qquad x(0) = 3$$

$$z\left[X(z) - \overset{3}{x(0)}\right] - 4X(z) = \frac{4z}{z-2} - \frac{4 \times 2z}{(z-2)^2}$$

$$X(z)(z-4) = 3z + \frac{4z}{z-2} - \frac{8z}{(z-2)^2} = z\left[\frac{3(z^2-4z+4) + 4(z-2) - 8}{(z-2)^2}\right]$$

$$X(z) = z\left[\frac{3z^2 - 12z + 12 + 4z - 8 - 8}{(z-2)^2(z-4)}\right] = z\left[\frac{3z^2 - 8z - 4}{(z-2)^2(z-4)}\right] = z\left[\frac{A}{z-2} + \frac{B}{(z-2)^2} + \frac{C}{z-4}\right]$$

$$\rightarrow A(z-2)(z-4) + B(z-4) + C(z-2)^2 = A(z^2 - 6z + 8) + B(z-4) + C(z^2 - 4z + 4) = 3z^2 - 8z - 4$$

$$\begin{cases} A + C = 3 \\ -6A + B - 4C = -8 \\ 8A - 4B + 4C = -4 \end{cases} \qquad \begin{matrix} A = 0 \\ B = 4 \\ C = 3 \end{matrix} \rightarrow X(z) = \frac{4z}{(z-2)^2} + \frac{3z}{z-4}$$

$$x(n) = 2n\,2^n + 3 \times 4^n$$

$$\boxed{x(n) = n\,2^{n+1} + 3 \times 4^n}$$

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7) Resuelve las sig. ecuaciones en diferencias de orden 2 utilizando la transformada Z:

a) $a_{n+2} + 3a_{n+1} + 2a_n = 3^n$ con $a_0 = 0 \wedge a_1 = 1$

$x(n+2) + 3x(n+1) + 2x(n) = 3^n$ con $x(0) = 0 \wedge x(1) = 1$

$z^2\left[X(z) - x(0) - \frac{x(1)}{z}\right] + 3z\left[X(z) - x(0)\right] + 2X_z = \frac{z}{z-3}$

$z^2 X(z) - z + 3z X(z) + 2X_z = \frac{z}{z-3}$

$X(z)\underbrace{(z^2 + 3z + 2)}_{(z+1)(z+2)} = \frac{z}{z-3} + z = \frac{z + z^2 - 3z}{z-3} = \frac{z^2 - 2z}{z-3}$

$X(z) = z\left[\frac{z-2}{(z-3)(z+1)(z+2)}\right] = z\left[\frac{A}{z-3} + \frac{B}{z+1} + \frac{C}{z+2}\right]$

$A(z^2+3z+2) + B(z^2-z-6) + C(z^2-2z-3) = z-2 \rightarrow \begin{cases} A+B+C = 0 \\ 3A - B - 2C = 1 \\ 2A - 6B - 3C = -2 \end{cases} \rightarrow \begin{aligned} A &= 1/20 \\ B &= 3/4 \\ C &= -4/5 \end{aligned}$

$X(z) = \frac{1}{20}\frac{z}{z-3} + \frac{3}{4}\frac{z}{z+1} - \frac{4}{5}\frac{z}{z+2} \rightarrow \boxed{x(n) = \frac{3^n}{20} + \frac{3}{4}(-1)^n - \frac{4}{5}(-2)^n}$ ✓

b) $a_{n+2} + 4a_{n+1} + 4a_n = 7$ con $a_0 = 1 \wedge a_1 = 2$

$x(n+2) + 4x(n+1) + 4x(n) = 7$ con $x(0) = 1 \wedge x(1) = 2$

$z^2\left[X(z) - x(0) - \frac{x(1)}{z}\right] + 4z\left[X(z) - x(0)\right] + 4X(z) = \frac{7z}{z-1}$

$z^2 X(z) - z^2 - 2z + 4zX(z) - 4z + 4X(z) = 7\frac{z}{z-1}$

$X(z)(z^2 + 4z + 4) = \frac{7z}{z-1} + z^2 + 6z = \frac{7z + z^3 - z^2 + 6z^2 - 6z}{(z-1)} = \frac{z^3 + 5z^2 + z}{z-1}$

$X(z) = z\left[\frac{z^2 + 5z + 1}{(z-1)(z+2)^2}\right] = z\left[\frac{A}{z-1} + \frac{B}{z+2} + \frac{C}{(z+2)^2}\right] = z\left[\frac{A(z^2+4z+4) + B\overbrace{(z-1)(z+2)}^{(z^2+z-2)} + C(z-1)}{(z-1)(z+2)^2}\right]$

$\begin{cases} A + B = 1 \\ 4A + B + C = 5 \\ 4A - 2B - C = 1 \end{cases} \rightarrow \begin{aligned} A &= 7/9 \\ B &= 2/9 \\ C &= 5/3 \end{aligned} \rightarrow X(z) = \frac{7}{9}\cdot\frac{z}{z-1} + \frac{2}{9}\frac{z}{z+2} + \frac{5}{3}\frac{z}{(z+2)^2}\cdot\frac{(-2)}{(-2)}$

$\boxed{x(n) = \frac{7}{9} + \frac{2}{9}\cdot(-2)^n - \frac{5}{6}n(-2)^n}$ ✓

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c) $x(n+2) - 4\,x(n+1) + 3\,x(n) = -2$ con $x(0)=7 \wedge x(1)=12$

$$z^2\left[X(z) - x(0) - \frac{x(1)}{z}\right] - 4z\left[X(z) - \overset{7}{x(0)}\right] + 3X(z) = \frac{-2z}{z-1}$$

$$z^2X(z) - 7z^2 - 12z - 4zXz + 28z + 3X(z) = \frac{-2z}{z-1}$$

$$X(z)\left(z^2-4z+3\right) = \frac{-2z}{z-1} + 7z^2 - 16z = \frac{-2z + 7z^3 - 7z^2 - 16z^2 + 16z}{z-1}$$

$$X(z) = \frac{7z^3 - 23z^2 + 14z}{(z-1)(z-3)(z-1)} = z\left[\frac{7z^2-23z+14}{(z-1)^2(z-3)}\right] = z\left[\frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z-3}\right]$$

$$\rightarrow A\overbrace{(z-1)(z-3)}^{z^2-4z+3} + B(z-3) + C(z^2-2z+1) = 7z^2 - 23z + 14 \rightarrow \begin{cases} A + C = 7 \\ -4A + B - 2C = -23 \\ 3A - 3B + C = 14 \end{cases} \quad \begin{aligned} A &= 5 \\ B &= 1 \\ C &= 2 \end{aligned}$$

$$X(z) = 5\frac{z}{z-1} + \frac{z}{(z-1)^2} + 2\frac{z}{z-3}$$

$$\boxed{x(n) = 5 + n + 2\times 3^n}$$

d) $x(n+2) - 4\,x(n+1) + 4\,x(n) = 4\cdot 3^n$ $x(0)=4 \wedge x(1)=14$

$$z^2\left[X(z) - \overset{4}{x(0)} - \frac{\overset{14}{x(1)}}{z}\right] - 4z\left[X(z) - \overset{4}{x(0)}\right] + 4X(z) = \frac{4z}{z-3}$$

$$z^2X(z) - 4z^2 - 14z - 4zX(z) + 16z + 4X(z) = \frac{4z}{z-3}$$

$$X(z)\left(z^2 - 4z + 4\right) = \frac{4z}{z-3} + 4z^2 - 2z = \frac{4z + 4z^3 - 12z^2 - 2z^2 + 6z}{z-3}$$

$$X(z) = z\left[\frac{4z^2 - 14z + 10}{(z-3)\,(z-2)^2}\right] = z\left[\frac{A(z^2-4z+4) + B(z^2-5z+6) + C(z-3)}{(z-3)\,(z-2)^2}\right] \rightarrow \begin{cases} A + B = 4 \\ -4A - 5B + C = -14 \\ 4A + 6B - 3C = 10 \end{cases} \quad \begin{aligned} A &= 4 \\ B &= 0 \\ C &= 2 \end{aligned}$$

$$X(z) = \frac{4z}{z-3} + \frac{2z}{(z-2)^2} \rightarrow \boxed{x(n) = 4\times 3^n + n2^n}$$

e) $x(n+2) - 6\,x(n+1) + 9\,x(n) = 5\times 2^n$ $x(0)=5 \wedge x(1)=13$

$$z^2\left[X(z) - x(0) - \frac{x(1)}{z}\right] - 6z\left[X(z) - x(0)\right] + 9X(z) = 5\frac{z}{z-2}$$

$$z^2X(z) - 5z^2 - 13z - 6zX(z) + 30z + 9X(z) = \frac{5z}{z-2}$$

$$X(z)\left(z^2 - 6z + 9\right) = \frac{5z}{z-2} + 5z^2 - 17z = \frac{5z + 5z^3 - 10z^2 - 17z^2 + 34z}{z-2}$$

$$X(z) = z\left[\frac{5z^2 - 27z + 39}{(z-2)\,(z-3)^2}\right] = z\left[\frac{A}{z-2} + \frac{B}{z-3} + \frac{C}{(z-3)^2}\right]$$

$$A(z^2-6z+9) + B(z^2-5z+6) + C(z-2) = 5z^2 - 27z + 39 \quad \begin{cases} A + B = 5 \\ -6A - 5B + C = -27 \\ 9A + 6B - 2C = 39 \end{cases} \rightarrow \begin{aligned} A &= 5 \\ B &= 0 \\ C &= 3 \end{aligned}$$

$$X(z) = 5\frac{z}{z-2} + \frac{3z}{(z-3)^2} \longrightarrow \boxed{x(n) = 5\times 2^n + n\,3^n}$$

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f) - $a_{n+2} = a_{n+1} + a_n$ con $a_0 = 0 \wedge a_1 = 1$

$$x(n+2) - x(n+1) - x(n) = 0 \qquad \text{con } x(0) = 0 \wedge x(1) = 1$$

$$z^2\left[X(z) - \overset{0}{x(0)} - \frac{\overset{1}{x(1)}}{z}\right] - z\left[X(z) - \overset{0}{x(0)}\right] - X(z) = 0$$

$$z^2 X(z) - z - z\,X(z) - X(z) = 0$$

$$X(z)\,(z^2 - z - 1) = z$$

$$X(z) = \frac{z}{z^2 - z - 1} = z\,\frac{1}{\left(z - \left(\frac{1+\sqrt5}{2}\right)\right)\left(z - \left(\frac{1-\sqrt5}{2}\right)\right)} = z\left[\frac{\overset{A = \frac{1}{\sqrt5}}{A}}{z - \left(\frac{1+\sqrt5}{2}\right)} + \frac{\overset{B = -\frac{1}{\sqrt5}}{B}}{z - \left(\frac{1-\sqrt5}{2}\right)}\right]$$

$$X(z) = \frac{1}{\sqrt5}\,\frac{z}{\left[z - \left(\frac{1+\sqrt5}{2}\right)\right]} - \frac{1}{\sqrt5}\,\frac{z}{\left[z - \left(\frac{1-\sqrt5}{2}\right)\right]} \rightarrow \boxed{x(n) = \frac{1}{\sqrt5}\left[\frac{1+\sqrt5}{2}\right]^n - \frac{1}{\sqrt5}\left[\frac{1-\sqrt5}{2}\right]^n} \checkmark$$

g) $a_{n+2} - a_n = \text{sen}\left(\frac{n\pi}{2}\right)$ con $a_0 = 1 \wedge a_1 = 1$

$$x(n+2) - x(n) = \text{sen}\left(\frac{n\pi}{2}\right) \qquad \text{con } x(0) = 1 \wedge x(1) = 1$$

$$z^2\left[X(z) - \overset{1}{x(0)} - \frac{\overset{1}{x(1)}}{z}\right] - X(z) = \frac{z\,\overset{1}{\text{sen}(\pi/2)}}{z^2 - 2z\,\underset{0}{\cos(\pi/2)} + 1} = \frac{z}{z^2+1}$$

$$z^2 X(z) - z^2 - z - X(z) = \frac{z}{z^2+1}$$

$$X(z)\,(z^2 - 1) = \frac{z}{z^2+1} + z^2 + z = \frac{z + z^4 + z^2 + z^3 + z}{z^2+1} = \frac{z^4 + z^3 + z^2 + 2z}{z^2+1}$$

$$X(z) = \frac{z^4 + z^3 + z^2 + 2z}{(z^2+1)(z^2-1)} = z\left[\frac{z^3 + z^2 + z + 2}{(z^4 - 1)}\right] = z\left[\frac{Az + B}{(z^2+1)} + \frac{C}{(z+1)} + \frac{D}{(z-1)}\right] =$$

$$\rightarrow A(z^3 - z) + B(z^2 - 1) + C(z^2+1)(z-1) + D(z^2+1)(z+1) =$$

$$= A(z^3 - z) + B(z^2 - 1) + C(z^3 - z^2 + z - 1) + D(z^3 + z^2 + z + 1) = z^3 + z^2 + z + 2$$

$$\begin{array}{l} z^3: \\ z^2: \\ z: \\ \#: \end{array}\left\{\begin{array}{l} A + C + D = 1 \rightarrow C + D = 1 - A \\ B - C + D = 1 \\ -A + C + D = 1 \rightarrow C + D = 1 + A \\ -B - C + D = 2 \end{array}\right. \Big\rangle \boxed{A = 0} \rightarrow \left\{\begin{array}{l} C + D = 1 \\ B - C + D = 1 \\ \cancel{C + D = 1} \\ -B - C + D = 2 \end{array}\right. \quad \begin{array}{l} B = -1/2 \\ C = -1/4 \\ D = 5/4 \end{array}$$

$$X(z) = -\frac12\,\frac{z}{z^2+1} - \frac14\,\frac{z}{z+1} + \frac54\,\frac{z}{z-1} \rightarrow \boxed{x(n) = -\frac12\,\text{sen}\left(\frac{n\pi}{2}\right) - \frac14(-1)^n + \frac54} \checkmark$$

Sylvina

⑧ Para las sig. ecuaciones en diferencias de orden 2 escriba la $X(z)$ y la $x(n)$ correspondiente a cada una:

a) $x(n+2) - 6x(n+1) + 9x(n) = 100(-2)^n$ con $x(0)=4 \wedge x(1)=-5$

$$z^2\left[X(z) - x(0) - \frac{x(1)}{z}\right] - 6z\left[X(z) - x(0)\right] + 9X(z) = 100 \cdot \frac{z}{z+2}$$

$$z^2 X(z) - 4z^2 + 5z - 6zX(z) + 24z + 9X(z) = \frac{100z}{z+2}$$

$$X(z)(z^2 - 6z + 9) = \frac{100z}{z+2} + 4z^2 - 29z = \frac{100z + 4z^3 + 8z^2 - 29z^2 - 58z}{z+2}$$

$$X(z) = z\left[\frac{4z^2 - 21z + 42}{(z+2)(z-3)^2}\right] = z\left[\frac{A}{z+2} + \frac{B}{z-3} + \frac{C}{(z-3)^2}\right]$$

$$A(z^2-6z+9) + B(z^2-z-6) + C(z+2) = 4z^2 - 21z + 42$$

$$\begin{cases} A+B = 4 \\ -6A - B + C = -21 \\ 9A - 6B + 2C = 42 \end{cases} \quad A=4,\ B=0,\ C=3$$

$$\boxed{X(z) = \frac{4z}{z+2} + \frac{3z}{(z-3)^2}} \longrightarrow \boxed{x(n) = 4(-2)^n + n3^n}$$

b) $x(n+2) - 4x(n+1) + 4x(n) = 4^{n+1} = 4 \times 4^n$ con $x(0)=1 \wedge x(1)=10$

$$z^2\left[X(z) - \overset{1}{x(0)} - \frac{\overset{10}{x(1)}}{z}\right] - 4z\left[X(z) - \overset{1}{x(0)}\right] + 4X(z) = 4 \cdot \frac{z}{z-4}$$

$$X(z)\underbrace{(z^2 - 4z + 4)}_{(z-2)^2} = \frac{4z}{z-4} + z^2 + \overbrace{10z - 4z}^{6z} = \frac{4z + z^3 - 4z^2 + 6z^2 - 24z}{z-4}$$

$$X(z) = z\left[\frac{z^2 + 2z - 20}{(z-4)(z-2)^2}\right] = z\left[\frac{A}{z-4} + \frac{B}{(z-2)} + \frac{C}{(z-2)^2}\right]$$

$$\rightarrow A(z^2 - 4z + 4) + B(z^2 - 6z + 8) + C(z-4) = z^2 + 2z - 20$$

$$\begin{cases} A+B = 1 \\ -4A - 6B + C = 2 \\ 4A + 8B - 4C = -20 \end{cases} \rightarrow \quad A=1,\ B=0,\ C=6$$

$$\boxed{X(z) = \frac{z}{z-4} + \frac{\overset{3\times 2}{6z}}{(z-2)^2}} \longrightarrow \boxed{x(n) = 4^n + 3 \cdot n2^n}$$

c) $a_{n+2} - 2a_{n+1} - 3a_n = 24 \times 3^n + 12 \times 5^n$ con $a_0 = 1 \wedge a_1 = 11$

$$x(n+2) - 2x(n+1) - 3x(n) = 24 \times 3^n + 12 \times 5^n$$

$$z^2\left[X(z) - \overset{1}{x(0)} - \frac{\overset{11}{x(1)}}{z}\right] - 2z\left[X(z) - \overset{1}{x(0)}\right] - 3X(z) = 24\frac{z}{z-3} + 12\frac{z}{z-5}$$

$$X(z)\underbrace{(z^2 - 2z - 3)}_{(z-3)(z+1)} = \frac{24z}{z-3} + \frac{12z}{z-5} + z^2 + \overbrace{11z - 2z}^{9z} = z\left[\frac{24}{z-3} + \frac{12}{z-5} + z + 9\right]$$

$z^2 - 8z + 15$

$$X(z) = z\left[\frac{24z - 120 + 12z - 36 + z^3 - 8z^2 + 15z + 9z^2 - 72z + 135}{(z-3)^2(z-5)(z+1)}\right] = z\left[\frac{z^3 + z^2 - 21z - 21}{(z-3)^2(z-5)(z+1)}\right]$$

$$= z\left[\frac{A}{z-3} + \frac{\overset{= -48/8}{B}}{(z-3)^2} + \frac{\overset{= 24/24}{C}}{z-5} + \frac{\overset{=0}{D}}{z+1}\right]$$

$A = 0$
$B = 6$
$C = 1$
$D = 0$

en z^3: $A + \overset{=1}{C} + \overset{=0}{D} = 1$, $A = 0$

$$\boxed{X(z) = \frac{6z}{(z-3)^2} + \frac{z}{z-5}} \longrightarrow \boxed{x(n) = 2n3^n + 5^n}$$

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EJERCICIOS PARA MARCAR LA RESPUESTA CORRECTA, tomados en finales:

(9) Marcar la única respuesta correcta de cada ítem:

(1) La transformada Z de:
$$x(n) = \begin{cases} 0 & \text{si } n \text{ es par} \\ 2(-3)^{-n} & \text{si } n \text{ es impar} \end{cases}$$ es:

✓ a) $X(z) = \dfrac{-6z}{9z^2-1}$ b) $X(z) = \dfrac{2z}{z+3}$ c) $X(z) = \dfrac{2z}{z^2+9}$ d) n.a.

$$X(z) = \sum_{k=0}^{\infty} 2 \cdot \left(\frac{1}{-3z}\right)^{2k+1} = -\frac{2}{3z} \sum_{k=0}^{\infty} \left(\frac{1}{9z^2}\right)^k = -\frac{2}{3z} \cdot \frac{1}{1-\frac{1}{9z^2}} = \frac{-2}{\cancel{3z}} \frac{\overset{3}{\cancel{9z^2}}}{9z^2-1} = \frac{-6z}{9z^2-1}$$

(2). La transformada Z de $x(n) = \begin{cases} 0 & \text{si } n \text{ es par} \\ 3 \times 2^n & \text{si } n \text{ es impar} \end{cases}$ es:

✓ a) $X(z) = \dfrac{3z}{z-2}$ b) $X(z) = \dfrac{3z}{z^2-4}$ c) $X(z) = \dfrac{3z^2}{z^2-4}$ d) $X(z) = \dfrac{6z}{z^2-4}$ e) n.a

$$X(z) = \sum_{k=0}^{\infty} 3\left(\frac{2}{z}\right)^{2k+1} = \frac{6}{z} \sum_{k=0}^{\infty} \left(\frac{4}{z^2}\right)^n = \frac{6}{z} \cdot \frac{1}{1-\frac{4}{z^2}} = \frac{6}{\cancel{z}} \frac{z^{\cancel{2}}}{(z^2-4)} = \frac{6z}{z^2-4}$$

(3) Sea $x(n) = n$ para $0 \le n \le 3$ $\wedge$ $x(n) = 0$ para $n \geq 4$ entonces su transformada Z es:

✓ a) $Z[x(n)] = \dfrac{z}{(z-1)^2}$ b) $Z[x(n)] = \dfrac{z+2z+3z}{(z-3)^2}$ c) $Z[x(n)] = \dfrac{z^2+2z+3}{z^3}$ d) n.a.

$$Z[x(n)] = X(z) = \sum_{n=1}^{3} \frac{n}{z^n} = \frac{1}{z} + \frac{2}{z^2} + \frac{3}{z^3} = \frac{z^2+2z+3}{z^3}$$

(4) Sean las sucesiones a_n y b_n entonces se cumple:

a) $Z[a_n \cdot b_n] = Z[a_n] \cdot Z[b_n]$

b) $Z[a_n^2] = Z[a_n]^2$

✓ c) $Z[a_n - k b_n] = Z[a_n] - k\, Z[b_n]$ x linealidad

d) $Z[a_{n+1}] = z\, Z[a_n] - a_0$

e) n.a.

Sylumna

⑤ La antitransformada Z de $X(z) = \dfrac{z(2z+3)}{z^2-7z+6}$ es:

a) $x(n) = 3(-6)^n + 1$

b) $x(n) = -3(-6)^n + (-1)^n$

✓ c) $x(n) = 3 \cdot 6^n - 1$

d) n.a.

$$X(z) = z\left[\frac{2z+3}{(z-6)(z-1)}\right] = z\left[\frac{A}{z-6} + \frac{B}{z-1}\right] \rightarrow \begin{cases} A+B = 2 \\ -A-6B = 3 \end{cases} \rightarrow \begin{matrix} A=3 \\ B=-1 \end{matrix}$$

$$X(z) = \frac{3z}{z-6} - \frac{z}{z-1} \rightarrow x(n) = 3 \times 6^n - 1$$

⑥ La antitransformada Z de $X(z) = \dfrac{12z}{z^2-9}$ es:

a) $x(n) = 12 \cdot 3^n$ b) $x(n) = 12[(3)^2]^n$ c) $x(n) = 12(-3)^n + 12 \times 3^n$ d) $x(n) = 4 \times 3^n$ si n es impar ✓

e) n.a.

$$X(z) = z\left[\frac{12}{(z-3)(z+3)}\right] = z\left[\frac{A}{z-3} + \frac{B}{z+3}\right] \rightarrow \begin{cases} A+B = 0 \\ 3A-3B = 12 \end{cases} \rightarrow \begin{matrix} A = 2 \\ B = -2 \end{matrix}$$

$$X(z) = \frac{2z}{z-3} - \frac{2z}{z+3} \rightarrow x(n) = 2 \times 3^n - 2 \times (-3)^n$$

si n es par $\rightarrow x(n) = 0$

si n es impar $\rightarrow x(n) =$

RESPUESTAS EJERCICIOS DE TRANSFORMADA Z:

☑ Ej. 1:

a) $X(z) = \dfrac{z}{z-\frac{1}{3}}\quad |z| > \dfrac{1}{3}$ b) $X(z) = \dfrac{z^2}{z^2-1}\ ;\ |z| > 1$ c) $X(z) = \dfrac{z^2}{z^2-16} + \dfrac{3z}{9z^2-1}\ ;\ |z| > 4$

d) $X(z) = \dfrac{5z}{z^2-1} + \dfrac{12\,z^2}{4z^2-1}\ ;\ |z| > 1$ e) $X(z) = \dfrac{2z^2+2z+3}{z^3}\ ;\ z \neq 0$

f) $X(z) = z\ln\left(1-\dfrac{1}{z}\right)\ ;\ |z| > 1$ g) $X(z) = e^{\frac{2}{z}}\ ;\ z \neq 0$

☑ Ej. 2:

$$X(z) = \frac{z^3+3z^2+2z+1}{z^4}$$

☑ Ej. 3:

a) $X(z) = \dfrac{8z^2-13z}{(z-2)(2z-1)};|z| > 2$ b) $X(z) = \dfrac{1}{z(z-1)}$ c) $X(z) = \dfrac{3z}{z-3}$

☑ Ej. 4:

a) FALSO. Es $|z| > 1/3$. b) VERDADERO. c) VERDADERO.

☑ Ej. 5:

a) $x(n) = 2 + 4^n$ b) $x(n) = \begin{cases} 0 & \text{si n es impar} \\ 2\,3^n & \text{si n es par} \end{cases}$ c) $x(n) = \left(-\dfrac{1}{3}\right)(-2)^n + \dfrac{1}{3}$

d) $a_n = \begin{cases} 0 & \text{si n es par} \\ \dfrac{2^n}{n!} & \text{si n es impar} \end{cases}$ e) $a_n = \dfrac{1}{(n+1)!}$ f) $a_n = \begin{cases} 0 & \text{si } n=0 \text{ o } n=1 \\ n-1 & \text{si } n \geq 2 \end{cases}$

g) $x(n) = -3 \bullet 2^n + 2.5\,n\,2^n + 3$ h) $x(n) = 2 \bullet 2^n - 2 - 3\,n$ i) $x(n) = x(n) = 4\,3^n + n\,2^n$

Ej. 6:

a) $a_n = -3 + 2 \bullet 5^n$ b) $a_n = -2 + 5 \bullet 2^n$ c) $a_n = 2 \bullet 3^n$

d) $a_n = 2 \bullet 2^n - 2 - 3n$ e) $a_n = 2n\,2^n + 3\,4^n$

Ej. 7:

a) $a_n = \frac{3}{4}(-1)^n - \frac{4}{5}(-2)^n + \frac{1}{20}3^n$ b) $a_n = \frac{2}{9}(-2)^n - \frac{5}{6}n\,(-2)^n + \frac{7}{9}$

c) $x(n) = 2 \bullet 3^n + 5 + n$ d) $x(n) = 4 \bullet 3^n + n \bullet 2^n$ e) $x(n) = 5 \bullet 2^n + n \bullet 3^n$

f) $a_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$ g) $a_n = \frac{5}{4} - \frac{1}{4}(-1)^n - \frac{1}{2}\,\mathrm{sen}\left(\frac{n\pi}{2}\right)$

Ej. 8:

a) $X(z) = \frac{z(4z^2-21z+42)}{(z+2)(z-3)^2} = \frac{4z}{z+2} + \frac{3z}{(z-3)^2} \Rightarrow x(n) = 4\,(-2)^n + n\,3^n$

b) $X(z) = \frac{z^3+2z^2-20z}{(z-2)^2(z-4)} \Rightarrow x(n) = 3 \bullet n \bullet 2^n + 4^n$

c) $X(s) = \frac{z(z^2-21)}{(z-3)^2(z-5)} = \frac{6z}{(z-3)^2} + \frac{z}{(z-5)} \Rightarrow x(n) = 2n\,3^n + 5^n$

Ej. 9:

1	2	3	4	5	6
a	d	c	c	c	d